When Everyone Upgrades: Replacement Complementarities and Endogenous Growth

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October 2026

Why Replace a Durable That Still Works?

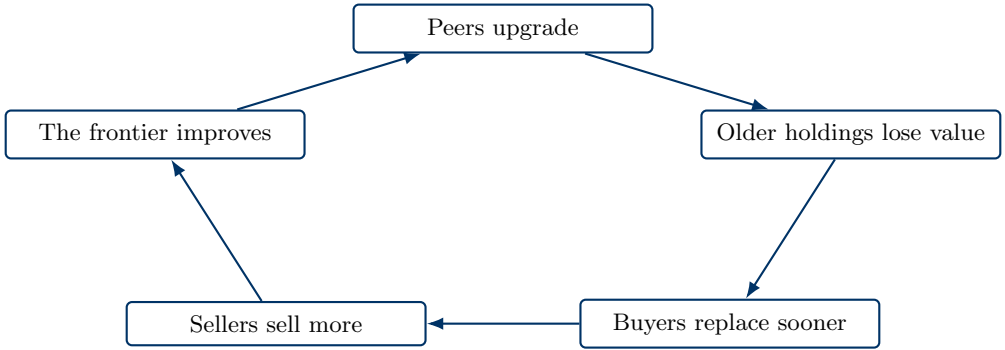
- **Households replace long before durables exhaust their lifecycle**
  - 13.8% of vehicles were acquired in the past year NHTS (2017)

- **What friends own matters**
  - a friend's purchase raises own phone demand Bailey et al. (2022)

- **Replacement purchases finance innovation**
  - replacement sales are the seller's market for product innovation

How do peer effects in durable replacement shape innovation and growth?

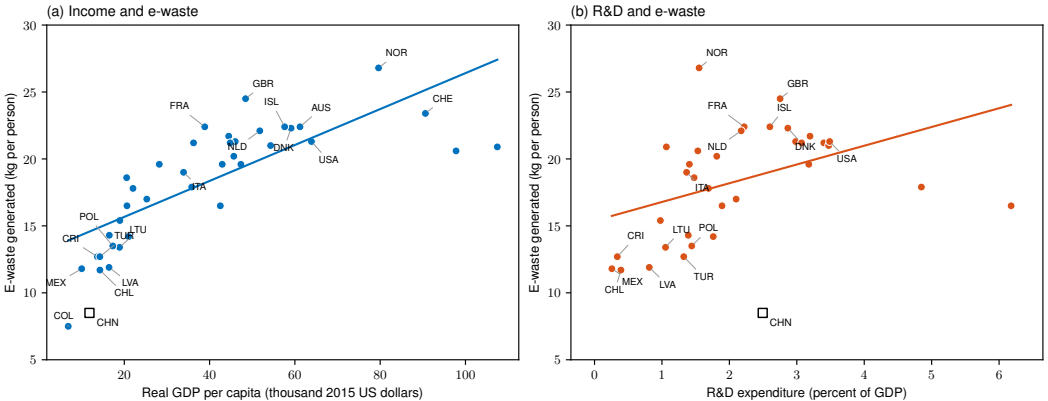
The Loop



Peer effects can sustain slow and fast replacement equilibria

Income, Replacement, and R&D Across Countries

- Generated e-waste per person rises with income and with R&D intensity (2022 OECD)



Sources: Global E-waste Monitor 2024 (Baldé et al.); World Development Indicators

Results Preview

- **Multiplicity with slow and fast replacement equilibria**
  - slow equilibrium: replacement every 14.7 years; growth 1.31%
  - fast equilibrium: replacement every 7.3 years; growth 2.99%

- **Higher production costs can remove the fast equilibrium**
  - the fast equilibrium disappears after a seller-margin loss above 0.55%
  - frontier growth falls from 3% to about 1.3% a year

Related Literature

Durable replacement: peers move the replacement threshold

Grossman & Laroque (1990); Caballero (1993); Eberly (1994)
Caplin & Leahy (2006); Goettler & Gordon (2011); Berger & Vavra (2015)
Bertolotti, Gavazza & Lanteri (2023)

Endogenous growth: replacement purchases support innovation

Grossman & Helpman (1991); Greenwood, Hercowitz & Krusell (1997),Perla & Tonetti (2014)
Benhabib, Perla & Tonetti (2021); Bertolotti & Lanteri (2024); Aghion et al. (2025)

Adoption with peers: peer comparisons sustain different growth rates

Katz & Shapiro (1985); Farrell & Saloner (1985); Lasry & Lions (2007)
Adlakha & Johari (2013); Bailey et al. (2022); Alvarez et al. (2023)

Model

Environment

- Continuous time; unit mass of buyers, each owns one durable of quality x

  $\dot{x} = -\delta x$, frontier price: $(\kappa + \pi^S)\bar{x}$, fixed cost replacement: $\chi \underline{x}$

- Product sublines $j \in [0, 1]$, CES across sublines: each seller charges a markup

  $p = \eta\kappa\bar{x}$, $\eta = \dfrac{\sigma}{\sigma - 1}$, $\pi^S \equiv (\eta - 1)\kappa$: margin per sale

- Each seller chooses frontier growth γ at cost $\omega\gamma^2\bar{x}$; profit $\pi^S \mathcal{J}\, \bar{x}$
  - $\mathcal{J}$: replacement flow, the mass of buyers that replace per unit time

Peer Comparisons

- Peer benchmark: the quality at which peers discard, and the quality they hold

  $M = (1-\zeta)\log \underline{x} + \zeta \log X, \qquad \zeta \in [0,1]:$ peer weight

- Flow service payoff at date t

  $$U(x,t) = \underbrace{\mathcal{A}(t)\,x}_{\text{own quality}} - \underbrace{\mathcal{B}(t)\,[\log x - M(t)]^2}_{\text{distance to peers}}$$

  – $\mathcal{B}$: curvature of the quadratic loss around the peer benchmark

- **Peers upgrade $\Rightarrow$ M rises $\Rightarrow$ older holdings lose value**

Buyers

- The buyer takes the paths of p, $\bar{x}$, $\underline{x}$, and M as given
- $V(x,t)$: maximum discounted service payoff net of replacement costs
  - x: holdings quality at date t $\quad$ ρ: discount rate

- The buyer chooses replacement dates $\{\tau_n\}_{n\geq 1}$ from date t onward

$$V(x,t) = \sup_{\{\tau_n\}} \Big\{ \int_t^\infty e^{-\rho(u-t)} U(x(u),u)\, du - \sum_{n\geq 1} e^{-\rho(\tau_n - t)} \big[p(\bar{x}(\tau_n),\tau_n) + \chi \underline{x}(\tau_n) \big] \Big\}$$

  - The sum includes every replacement purchase

- Holdings depreciate between replacements and reset at each replacement

$$\dot{x}(u) = -\delta x(u) \quad \text{between replacements,} \qquad x(\tau_n) = \bar{x}(\tau_n)$$

Sellers

- The seller takes the path of replacement flow $\mathcal{J}(u)$ as given

- $S(\bar{x}, t)$: maximum discounted profit from frontier quality $\bar{x}$ at date t

$$S(\bar{x}, t) = \sup_{\gamma(\cdot)\geq 0} \int_t^\infty e^{-\rho(u-t)} \left[\pi^S \mathcal{J}(u) - \omega\gamma(u)^2\right] \bar{x}(u)\, du$$

- Innovation raises frontier quality

$$\dot{\bar{x}}(u) = \gamma(u)\bar{x}(u), \qquad \bar{x}(t) = \bar{x}$$

  - Higher frontier quality raises profit per sale and the cost of innovation

Symmetric Equilibrium

Definition. Values, policies, prices, and a distribution of holdings form an equilibrium if: Buyers share a replacement policy, and sellers share prices and an innovation policy.

1. Buyers choose replacement dates that attain $V(x,t)$
2. Sellers set markup prices and choose innovation that attains $S(\bar{x},t)$
3. Sales equal replacement purchases, with flow $\mathcal{J}(t)$
4. The density of holdings $f(x,t)$ follows depreciation and replacement
5. Average holdings quality and the peer benchmark satisfy

$$X(t) = \int_{\underline{x}(t)}^{\bar{x}(t)} x f(x,t)\, dx, \qquad M(t) = (1-\zeta)\log \underline{x}(t) + \zeta \log X(t)$$

Balanced Growth

Balanced-Growth Equilibrium

Definition. A balanced-growth equilibrium is an equilibrium with the following properties:

- Frontier quality and the replacement threshold grow at a constant rate $g \in (0, \rho)$

  $$\bar{x}(t) = \bar{x}(0)e^{gt}, \qquad \underline{x}(t) = \underline{x}(0)e^{gt}$$

- The distribution of holdings relative to the replacement threshold is constant

- Value functions are stationary in quality units
  - buyer value in threshold units, at each relative holdings quality
  - seller value per unit of frontier quality

- Replacement flow $\mathcal{J}$ is constant and each seller chooses $\gamma = g$

Normalization

- Holdings, frontier quality, and average holdings quality relative to the threshold

$$q \equiv \log\left(\frac{x}{\underline{x}}\right), \qquad q^* \equiv \log\left(\frac{\bar{x}}{\underline{x}}\right), \qquad Q \equiv \log\left(\frac{X}{\underline{x}}\right), \qquad \Delta \equiv \delta + g$$

  - Δ: drift of holdings toward replacement

- Stationary service payoff: $\mathcal{A} = A$ and $\mathcal{B} = B\underline{x}$

$$u(q, Q) \equiv A(e^q - 1) - B(q - \zeta Q)^2$$

- Normalized buyer value v and seller value s

$$v(q) \equiv \frac{V(x,t)}{\underline{x}(t)} - \frac{A}{\rho - g}, \qquad s \equiv \frac{S(\bar{x},t)}{\bar{x}}$$

  - v: buyer value in threshold units, net of the constant service payoff A
  - s: seller value per unit of frontier quality

Buyers: Optimal Stopping

- Given g, Q, and q^*, the buyer chooses the waiting time $\tau \geq 0$

$$v(q) = \sup_{\tau \geq 0} \left\{ \int_0^\tau e^{-(\rho-g)t} u(q - \Delta t, Q)\, dt + e^{-(\rho-g)\tau} \left[v(q^*) - (\kappa + \pi^S) e^{q^*} - \chi \right] \right\}$$

- Keeping holdings is optimal on $q \in (0, q^*]$

$$(\rho - g) v(q) = u(q, Q) - \Delta v'(q)$$

- Replace at $q = 0$ and reset to q^*: value matching and smooth pasting

$$v(q^*) - v(0) = (\kappa + \pi^S) e^{q^*} + \chi, \qquad v'(0) = 0$$

Buyers: The Distribution of Holdings

- Holdings drift from q^* to 0 at rate Δ between replacements
  - A replacement cycle lasts q^*/Δ years

- On a balanced-growth path, q has a uniform density on $[0, q^*]$

$$\mathcal{J} = \frac{\Delta}{q^*} = \frac{\delta + g}{q^*}$$

  - Replacement flow equals the inverse of the replacement cycle length

- Log average quality of holdings relative to the threshold

$$Q = \log\left(\frac{1}{q^*}\int_0^{q^*} e^q \, dq\right) = \log\left(\frac{e^{q^*} - 1}{q^*}\right)$$

Sellers: The HJB on a Balanced-Growth Path

- The Hamilton–Jacobi–Bellman (HJB) equation for normalized seller value is
$$\rho s = \pi^S \mathcal{J} + \max_{\gamma \geq 0} \{\gamma s - \omega \gamma^2\}$$

- Innovation is optimal at $\gamma = s/(2\omega)$ and equilibrium requires $\gamma = g$
$$s = 2\omega g, \qquad (\rho - g)s = \pi^S \mathcal{J} - \omega g^2$$

- Replacement sales support growth along the seller curve
$$q_S(g) = \frac{\overbrace{\pi^S(\delta + g)}^{\text{seller's gross market}}}{\underbrace{\omega\, g\, (2\rho - g)}_{\text{cost of sustaining growth}}}$$

- **Faster replacement ⇒ larger market ⇒ faster growth**
  - faster growth makes existing holdings obsolete sooner

Multiplicity

Peers Make Replacement a Complement

- Buyer residual: net benefit of replacing at the candidate range
  $F_B(q^*, g) \equiv v(q^*) - v(0) - (\kappa + \pi^S)e^{q^*} - \chi$

- **Proposition.** The response to average peer quality is
  $\frac{\partial F_B}{\partial Q} = B\,\zeta\,\Xi_1(q^*, \Delta, \beta) > 0 \qquad \text{whenever } B\zeta > 0$

- A higher peer benchmark raises the cost of keeping older holdings
  - buyers replace sooner $\Rightarrow$ more sales $\Rightarrow$ more innovation

One Equation in Frontier Growth

- Substitute the seller curve into the buyer residual: affine in the peer weight

  $$\Psi(g;\zeta) \equiv F_B\big(q_S(g), g\big) = S_0(g) + \zeta\, S_1(g), \qquad S_1(g) > 0$$

- Required peer weight: the ζ at which buyers are indifferent on the seller curve

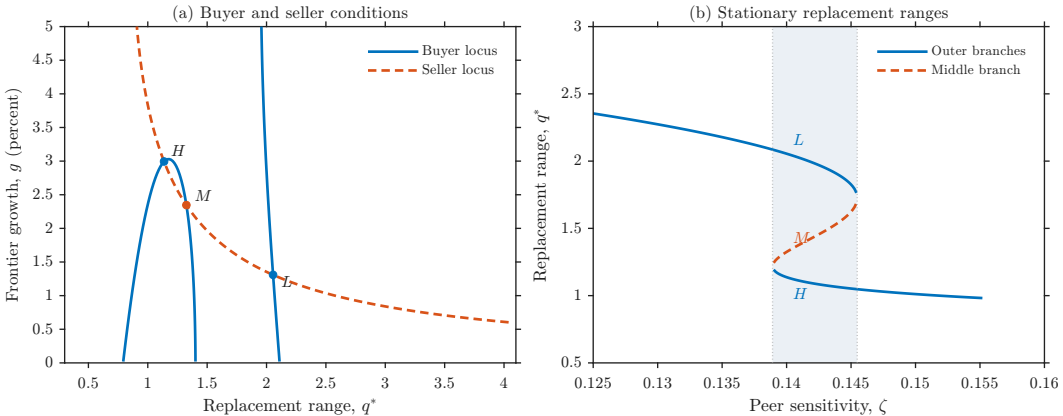
  $$Z(g) \equiv -\frac{S_0(g)}{S_1(g)}, \qquad \text{stationary equilibria: } Z(g) = \zeta$$

- Z monotone $\Rightarrow$ at most one equilibrium
- **A turning point in Z can support several equilibria**

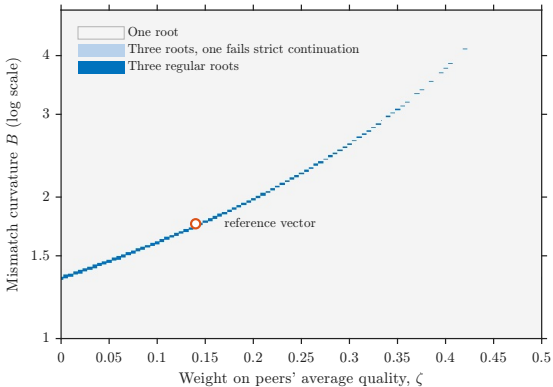
▸ Theorem

Three Equilibria

- Buyer and seller loci cross three times; the window in ζ is shaded in (b)



Region of Multiple Equilibria



- Three equilibria coexist
  - for a region of ζ and B
- Higher ζ requires higher B
- Other parameters held fixed

▸ Preference sensitivity

Quantification

Calibration: One Moment per Parameter

Moment or source	Parameter	Value
Set directly		
annual discount rate	ρ	0.080
car quality loss, 11.9% a year; Attanasio et al. (2022)	δ	0.127
aggregate markup; De Loecker et al. (2020)	η	1.61
value lost at adjustment; Berger & Vavra (2015)	$\chi/(\kappa + \pi^S)$	0.0525
Matched in the fast equilibrium		
relative durable price decline, 2.99% a year; BEA	ω	7.31
annual vehicle acquisitions, 13.8%; NHTS	$\kappa + \pi^S$	0.545
weekly peer response, 0.041 pp; Bailey et al. (2022)	B	1.75
coexistence region $[0.1375, 0.1425]$	ζ	0.140

▸ Across peer weights

Replacement, Innovation, and Welfare

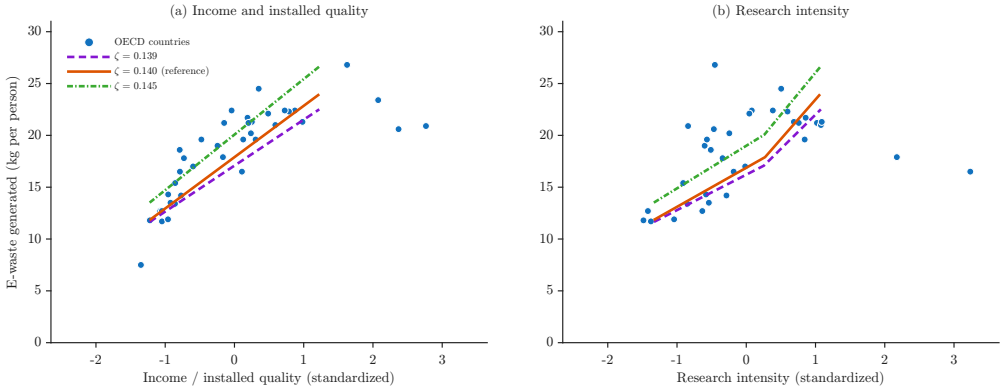
Equilibrium	g (%)	q^*	$\mathcal{J}$ (%)	R&D/sales (%)	Age (years)	Welfare W
L: slow replacement	1.31	2.06	6.80	3.38	14.7	2.31
M: middle	2.35	1.32	11.33	6.51	8.8	5.25
H: fast replacement	2.99	1.14	13.78	8.72	7.3	7.03

- Same preferences, costs, and physical depreciation in all three equilibria
- Economic obsolescence alone generates the different replacement ages
- Welfare is highest in H: older holdings in L deliver fewer services

▸ Holdings

Countries as Mixtures of Equilibria

- Two subline groups per country; each is in equilibrium H with probability p_H
- Fit the mean and standard deviation of e-waste



Fitted: $p_H = 0.54$, effective mass $m = 174$ kg. Untargeted tail means: 11.8 vs 11.9 and 24.0 vs 23.3 kg

Experiment: Rising Production Costs

Experiment: Memory Costs and Replacement

- **AI data centers draw memory supply toward servers**
  - PC and phone component costs rise TrendForce (2026)

- **2026 forecasts** Gartner (2026)
  - shipments: PCs -10.4%, smartphones -8.4%
  - PC lifetimes: $+15\%$ for business buyers, $+20\%$ for consumers

- **Experiment: higher production costs and lower seller margins**

Can higher production costs permanently slow innovation?

Experiment Design

- Unexpected permanent rise in unit cost κ, with a fall in the markup η

  $$\pi^S \;\to\; (1-\ell)\,\pi^S, \qquad \kappa \;\to\; \kappa + 4\,\ell\,\pi^S$$

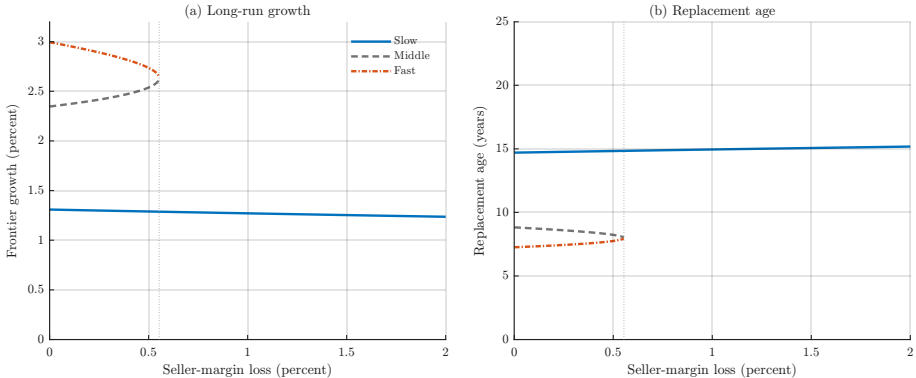
  - one quarter lowers the seller margin; three quarters raise the price

- Seller-margin losses ℓ of 0.25, 0.5, and 10 percent
  - at 10%: η falls from 1.61 to 1.44, σ rises from 2.6 to 3.3

- **Two forces: buyers wait longer, and each sale earns less**
  - both shrink the seller's return to innovation

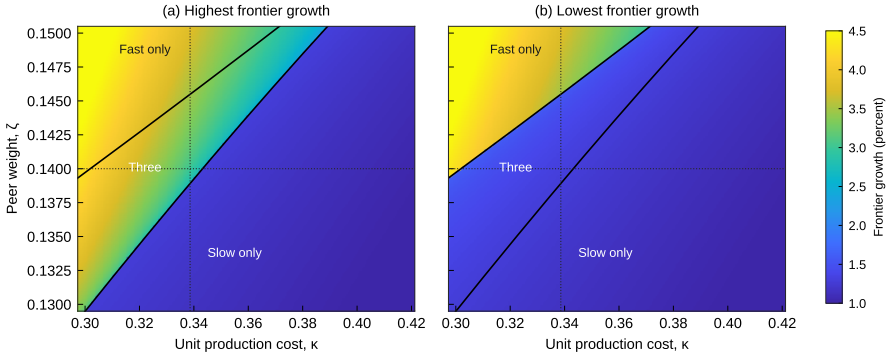
Higher Costs Can Remove the Fast Equilibrium

- 0.5% margin loss: growth 2.99% → 2.74%; replacement age 7.3 → 7.75 years
- **Above 0.55%: the slow equilibrium remains**
- 10% margin loss: replacement age 16.6 years; growth 1.03%



Higher Costs Across Peer Weights

- Highest and lowest stationary growth; black curves bound three equilibria
- Higher cost: the two coincide at slow replacement
- Higher peer weight: the fast equilibrium survives larger cost increases



Takeaways

1. Peer effects sustain multiple replacement and growth equilibria

2. Equilibria features either slow or fast replacement

3. A permanent cost increase lowers replacement and frontier growth

Backup: Multiplicity Theorem

Multiplicity of stationary equilibria

$B > 0$; staggered holdings. At ζ_F, the roots on K are g_F and a regular root g_B.

$$Z(g_F) = \zeta_F, \qquad Z'(g_F) = 0, \qquad Z''(g_F) \neq 0.$$

Both roots lie in the interior of K; strict stopping and interiority hold nearby.

$$\#\mathcal{E}_K(\zeta) = \begin{cases} 3, & (\zeta - \zeta_F)/Z''(g_F) > 0, \\ 1, & (\zeta - \zeta_F)/Z''(g_F) < 0, \end{cases} \qquad \text{for } \zeta \text{ sufficiently close to } \zeta_F.$$

- Slow equilibrium: infrequent replacement and slow innovation
- Fast equilibrium: frequent replacement and rapid innovation

Back

Backup: Calibration Across Peer Weights

			L		M		H	
ζ	B	$\kappa + \pi^S$	g (%)	Age	g (%)	Age	g (%)	Age
0.1375	1.787	0.516	1.20	15.9	2.77	7.7	2.99	7.3
0.1400	1.755	0.545	1.31	14.7	2.35	8.8	2.99	7.3
0.1425	1.724	0.573	1.59	12.4	1.80	11.0	2.99	7.3

- At each ζ: B matches the one-week peer response, ω the seller condition, $\kappa + \pi^S$ the buyer condition
- Welfare is highest in H at each ζ

Back

Backup: Quality of Holdings Relative to the Frontier

- Replacement thresholds: $r = x/\bar{x} \approx 0.13$ in L and 0.32 in H
- Holdings above $r = 0.8$: 11% of buyers in L; 20% in H

